AN UPWIND MULTIGRID ALGORITHM FOR CALCULATING FLOWS ON UNSTRUCTURED GRIDS

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Abstract

An algorithm is described that calculates inviscid, laminar, and turbulent flows on triangular meshes with an upwind discretization. A brief description of the base solver and the multigrid implementation is given, followed by results that consist mainly of convergence rates for inviscid and viscous flows over a NACA four-digit airfoil section. The results show that multigrid does accelerate convergence when the same relaxation parameters that yield good single-grid performance are used; however, larger gains in performance can be realized by doing less work in the relaxation scheme.

Introduction

Recently, unstructured-grid solvers have been used to extensively analyze multielement airfoils. One particular study used an implicit upwind method with a one-equation turbulence model. An attempt was made to assess grid convergence, but the computer resources necessary to compute on a sufficiently dense grid to achieve grid independence of near-surface velocity profiles were prohibitive.[1]

The current work is an attempt to improve the efficiency of the above-mentioned method by implementing a multigrid algorithm similar to that used by Mavriplis[2], who used independently generated grids as opposed to deriving coarser grid levels from the finest grid. A description of the base flow solver is presented first, followed by a brief description of the multigrid implementation. Finally, a few results are given.

Flow Solver

The base solver is a nodal scheme that solves either the Euler or Reynolds-averaged Navier-Stokes equations in integral form. The triangular-mesh formulation of Roe’s flux difference splitting, similar to that of Barth and Jesperson[3] is used to discretize the convective terms. A Galerkin formulation is used for the diffusive terms in the Navier-Stokes equations.

The solution is advanced in pseudotime with a first-order backward Euler formulation. The resulting linear system of equations is solved iteratively at each step in pseudotime with a prescribed number of subiterations of a red-black Gauss-Seidel scheme.

The one-equation models of Baldwin and Barth and Spalart and Allmaras are available for computing turbulent flows. Both models are described in reference [5].

Multigrid Algorithm

The algorithm is implemented with a mix of C and FORTRAN. The main driver consists of C code that handles data management and calls FORTRAN subroutines to perform the calculations, which allows dynamic memory allocation and a flexible data structure. The implementation of the multigrid algorithms also simplified.

In his multigrid tutorial, Briggs[4] presents a recursive algorithm for a family of multigrid cycles. By specifying a cycle index, either a V or W cycle can be specified. By coding the multigrid algorithm in C, the algorithm could be used directly in its recursive form to result in a very compact code segment.

In the restriction operation from the fine grid to the coarse grid, flow variables are transferred via linear interpolation with the three vertices of the fine grid cell that encloses each coarse grid node. The residual is transferred by distributing the residual at each fine grid node to the three vertices of the coarse grid cell that encloses it. Prolongation of the coarse grid correction is performed with linear interpolation similarly to the restriction operation.

Results

The first case presented is a transonic flow about a NACA 0012 airfoil. The finest grid level consists of 16,640 nodes of which 256 are on the airfoil surface. Figure 1 shows the L2 norm of the residual versus the number of cycles for a W cycle. For these cases, the CFL number and the number of subiterations are 200 and 20, respectively, and are the same for the multigrid and single-grid runs. These values yield good performance for single-grid cases. The asymptotic convergence rate for the multigrid cases is approximately 0.72, and the rate for the single-grid case is approximately 0.98.

Figure 2 shows the L2 norm of the residual versus the computer time, which is a truer measure of the benefit of multigrid. Note that if the identical parameters are used for the relaxation algorithm, then some gain in performance results; however, multigrid allows the use of much fewer subiterations, which results in a significant gain in performance.

A case of laminar flow about a NACA 0012 airfoil is shown in Fig. 3. The finest grid level consists of 16,116 nodes of which 256 are on the airfoil surface. The use of multigrid again shows improved convergence.

Preliminary runs have been made for a turbulent case on a NACA 0012, and the results are presented in Fig. 4. The finest grid level consists of 12,431 nodes of which 202 are on the airfoil surface. These runs were made by calculating the turbulence model on the finest grid after each multigrid cycle and restricting the turbulence variable to the coarser grids in the cycle. "Frozen Spalart" indicates that the eddy viscosity obtained with the Spalart-Allmaras turbulence model was held constant after 40 cycles. Once again, multigrid provides an appreciable improvement in convergence; however, work still needs to be done to determine why convergence slows down for the turbulent cases.

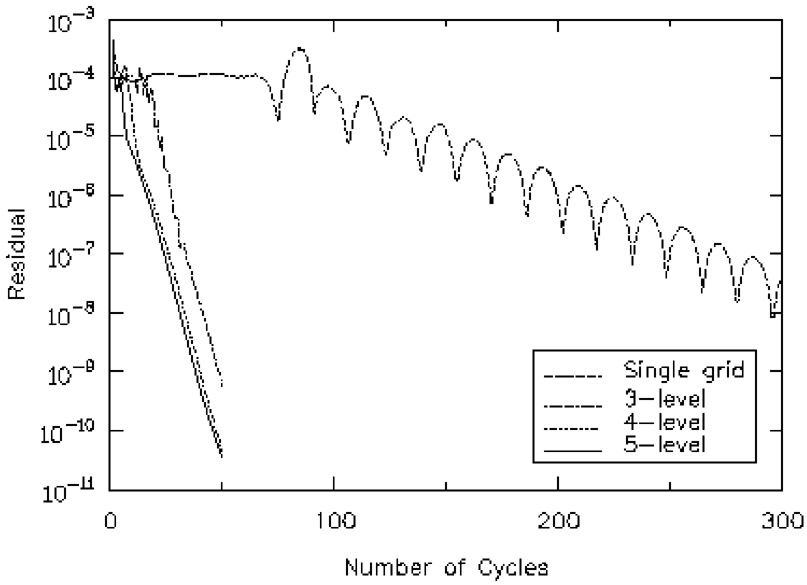

Figure 1. Effect of number of grid levels on W cycle performance for flow over a NACA 0012 airfoil, $M_{\&infty;}$=0.8, α =1.25&degrees;.

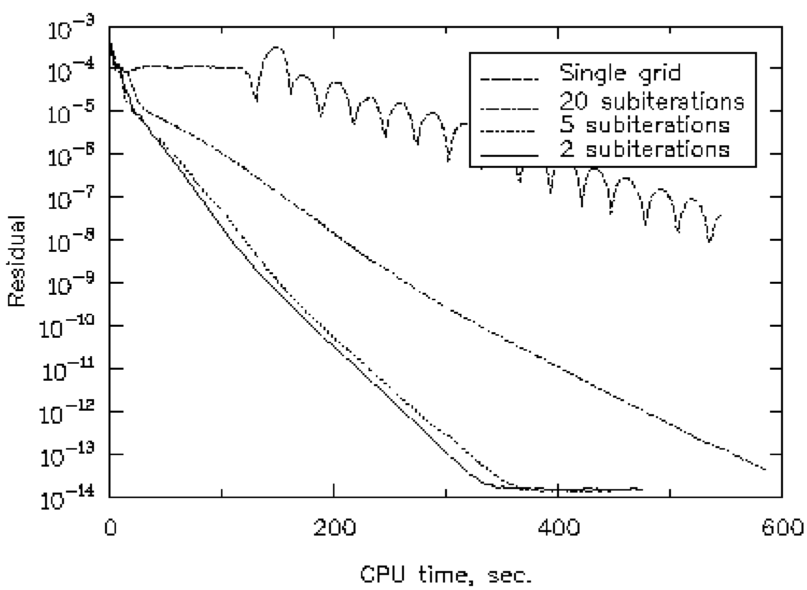

Figure 2. Effect of relaxation scheme parameters on convergence rate of five-level W cycle for NACA 0012 airfoil, $M_{\&infty;}$=0.8 , α =1.25&degrees;.

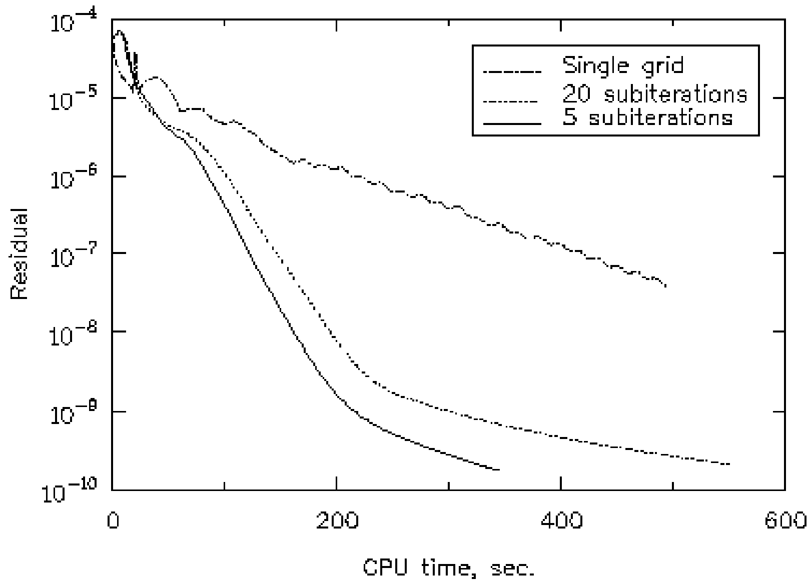

Figure 3. Effect of relaxation scheme parameters on performance of four-level W cycle for laminar flow over NACA 0012 airfoil, $M_{\&infty;}$=0.5, α =3&degrees;, Re=5000.

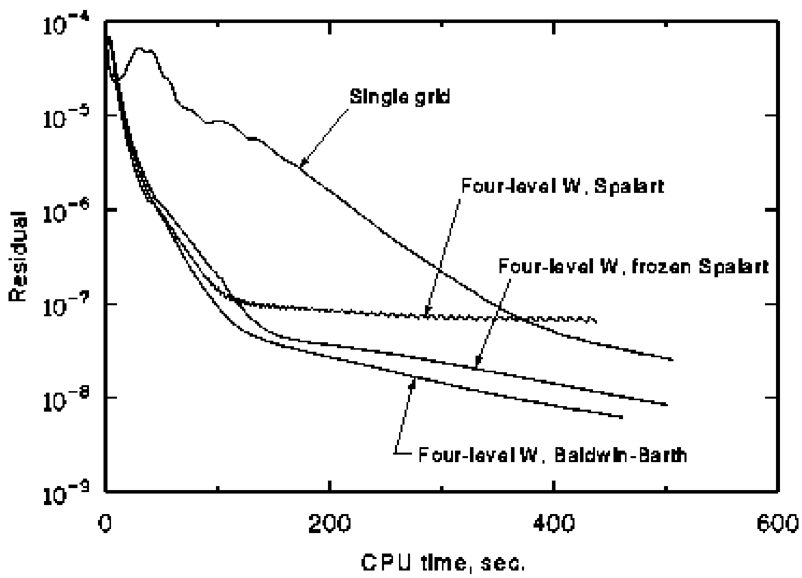

Figure 4. Effect of relaxation scheme parameters on performance of four-level W cycle on turbulent flow over NACA 0012 airfoil, $M_{\&infty;}$=0.7, α =1.49&degrees;, Re=9,000,000.

References

1. Anderson, W. K., and Bonhaus, D. L., "Navier-Stokes Computations and Experimental Comparisons for Multielement Airfoil Configurations," AIAA Paper 93-0645, Jan. 1993.
2. Mavriplis, D. J., Jameson, A., and Martinelli, L., "Multigrid Solution of the Navier-Stokes Equations on Triangular Meshes," NASA CR-181786, Feb. 1989.
3. Barth, T. J., and Jesperson, D. C., "The Design and Application of Upwind Schemes on

Unstructured Meshes," AIAA Paper 89-0366, Jan. 1989.
4. Briggs, W. L., *A Multigrid Tutorial*, Society for Industrial and Applied Mathematics, 1987.
5. Rumsey, C. L. and Vatsa, V. N., "A Comparison of the Predictive Capabilities of Several Turbulence Models Using Upwind and Central-Difference Computer Codes," AIAA Paper 93-0192, Jan. 1993.

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